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STATE ESTIMATION IN SIMULTANEOUS LOCALIZATION AND MAPPING FOR NONLINEAR PROCESSES WITH EXTENDED KALMAN FILTER

Abstract

This paper presents the use of Extended Kalman Filter (EKF) in the state estimation problem for nonlinear processes. Special focus is put on utilizing it in the algorithm of Simultaneous Localization and Mapping (SLAM). A number of simulations with a typical SLAM scenario were performed to examine the EKF properties. It is shown that the EKF algorithm can estimate the state accuracy in a typical SLAM scenario with relatively small error.

1. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is a fundamental problem of autonomous navigation that has been an active area of much scientific research [1][2]. In the classical approach it focuses on building a consistent spatial representation of an unknown environment with a moving robot, while simultaneously determining the location within that representation [3]. SLAM relies on incremental position estimation, as errors accumulate over time. Therefore acquiring a consistent spatial representation makes it a challenging problem.

The task of building a map and localizing a robot within that map can be understood as a state estimation problem [4][5]. The process can be represented by a mobile robot that performs measurements around the environment. Whereas, the state that needs to be estimated is the map and the robot location within its boundaries. The robot can gather information about the environment with a sensor. Nevertheless, it is necessary to filter out the noise, as every measurement is related with an error. Therefore a state estimation procedure needs to be performed to determine the best state that describes the process at any given time.

Most of SLAM algorithms assume the lack of a global positioning sensor and instead utilize local sensors to estimate the location of the robot and the map of the environment [6]. There are many different types of devices that are being used to measure the environment and gather information. Especially popular are laser scanners, as they can provide very accurate measurements. Despite the popularity and accuracy they have one major drawback, namely their high price. In fact, this makes them unpractical for certain applications. A review of the recent literature [7][8] shows that CCD cameras have gained more popularity within the SLAM community, as they are relatively cheap and can provide a wide amount of useful features.

2. EXTENDED KALMAN FILTER

Kalman Filter (KF) is used to tackle the problem of estimating the state of a process that can be expressed by a linear differential equation. But in many cases the process or the relation between the state and the measurement does not follow a linear model. Therefore it is necessary to use an Extended Kalman Filter (EKF), as it will linearize the process [9].

The EKF uses a nonlinear function *f*, which takes as the input: the previous state, the control and the error. In comparison a linear combination is being used in the KF. The process model can be described by the equation:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{2.1}$$

where: *x* – state vector,
f – nonlinear function,
u – control vector,
w – error vector.

The presence of the *f* function is due to the fact that the system includes a nonlinear dynamic. If neither the speed nor the acceleration is modelled, then the process control equation will remain linear. Consequently it will only account for the odometry, as the measure for the robot dynamics. Eventually no linearization is required, so the process control equation remains linear:

$$x_k = A x_{k-1} + B u_{k-1} + w_k \tag{2.2}$$

where: *A* – state transition matrix,
B – control transition matrix.

In the observation model it is common to find a nonlinear function *h* that takes both the state of the process and the noise as arguments. For simplicity it can be defined as a linear combination of a nonlinear function of the state and the noise:

$$z_k = h(x_k) + v_k \tag{2.3}$$

where: z – measurement vector,
h – nonlinear function,
v – measurement error.

It can be noted that the random variable *x* is no longer Gaussian after the nonlinear transformation *(h(x))*, as a result the measurement *z* will also not remain Gaussian.

The EKF can be divided in two separate stages: the time update and the measurements update. The first one can be defined by the equations:

$$\hat{x}_{k+1} = Ax_k + Bu_k \tag{2.4}$$

$$\hat{P}_{k+1} = AP_k A^T + Q \tag{2.5}$$

where: $\hat{P}_{k+1}$ – predicted estimate covariance matrix,

Q – process error covariance matrix.

Consequently the latter stage is being also defined as a set of equations:

$$K_{k+1} = \frac{\hat{P}_{k+1} J_H^T}{J_H \hat{P}_{k+1} J_H^T + R} \tag{2.6}$$

$$x_{k+1} = \hat{x}_{k+1} + K_{k+1}(z_{k+1} - h(\hat{x}_{k+1})) \tag{2.7}$$

$$P_{k+1} = (I - K_{k+1} J_H)\hat{P}_k \tag{2.8}$$

where: K_{k+1} – Kalman gain,

J_H – Jacobian matrix of the function f.

R – measurement error covariance matrix.

In the time update step the a priori state is estimated with the use of equation (2.4) and the covariance error with equation (2.5). The measurement update step calculates the Kalman gain, as shown in equation (2.6) and then uses it to improve the a posteriori estimation together with the measurement information in equation (2.7). Finally the a posterior covariance error is calculated by equation (2.8). It needs to be noted that in the EKF to approximate the a priori measurement the *h* function is used. Additionally a Jacobian J_H is being introduced in calculating the Kalman Gain equation.

The process control equation might be governed by a nonlinear function. However, only the measurement needs to be linearized, as linear models are present for the dynamics of the process. The a priori measurement estimation is computed by truncating the error v_k in the measurement difference equation and evaluating the function *h* on the a priori state estimate:

$$\hat{z}_k \approx h(\hat{x}_k) \tag{2.9}$$

Once the a priori estimate can be computed, the a posterior estimate is linearized, as a first order approximation based on the Jacobian evaluated at the a priori state estimate:

$$z_k \approx \hat{z}_k + J_H(x_k - \hat{x}_k) + v_k \tag{2.10}$$

where: $\hat{z}_k$ – a priori measurement estimate.

3. EXPERIMENT AND RESULTS

Presented algorithm was tested in a simulated environment. For this purpose implementation of EKF was written in MATLAB. The task of the robot was to move around an artificial plane, while simultaneously observing the distance and bearing to certain landmarks. Initial parameters like the number of landmarks and the control sequence were specified by the user in a MATLAB GUI form. In assumed scenario the robot can follow the specified control only with some uncertainty. For this reason the goal is to estimate the original trajectory as best as possible with the given control sequence and observations. Furthermore, additional estimations are also made to the positions of the landmarks.

The state of the robot can be described by its position (x_I, y_I) on the plane and a heading parameter θ, where all measured are relative to the original pose of the robot. The robot is controlled by specifying the pair of left and right rotational speed (φ_l, φ_r). State of the robot evolves according to:

$$x_I = \cos(\theta_I)x_R - \sin(\theta_I)y_R \tag{3.1}$$

$$y_I = \sin(\theta_I)x_R + \cos(\theta_I)y_R \tag{3.2}$$

$$\theta_I = \frac{r\varphi_r}{d} - \frac{r\varphi_l}{d} \tag{3.3}$$

where: x_R, y_R – local reference frame,
x_I, y_I – global reference frame,
θ_I – heading parameter,
φ_l, φ_r – rotational speed,
d – distance between the wheels,
r – radius of the wheels.

The robot observation is based on the distance and the viewing angle of the landmarks $L(x_L, y_L)$. If the robot is close to the landmark an observation $z_k = (R_L, \beta_L) + v_k$ can be performed, where:

$$R_L = \sqrt{(x_L - x_I)^2 + (y_L - y_I)^2} \tag{3.4}$$

$$\beta_L = \tan^{-1}\left(\frac{y_L - y_I}{x_L - x_I}\right) - \theta_I \tag{3.5}$$

The algorithm form Section 3 is applied in further calculations to estimate the position of the robot. It is assumed that while the robot is moving along a path all of the landmarks remain stationary. Additionally the range of the measuring sensor is limited to a certain constant distance. Figure 1 shows the results of a finished simulation run.

As shown in Table 3.1 the accuracy of the state estimation strongly depends on the number of landmarks. The results of the state estimation error have decreased after incorporating additional landmarks and in the most optimistic case reached less than 15%. In comparison, some state of the art systems achieve less than 5 % error [10], but that depends on many factors. Moreover, it is suspected that further incorporation of additional landmarks might improve overall results. Unfortunately performed calculations can become computationally demanding, as more and more landmarks are added.

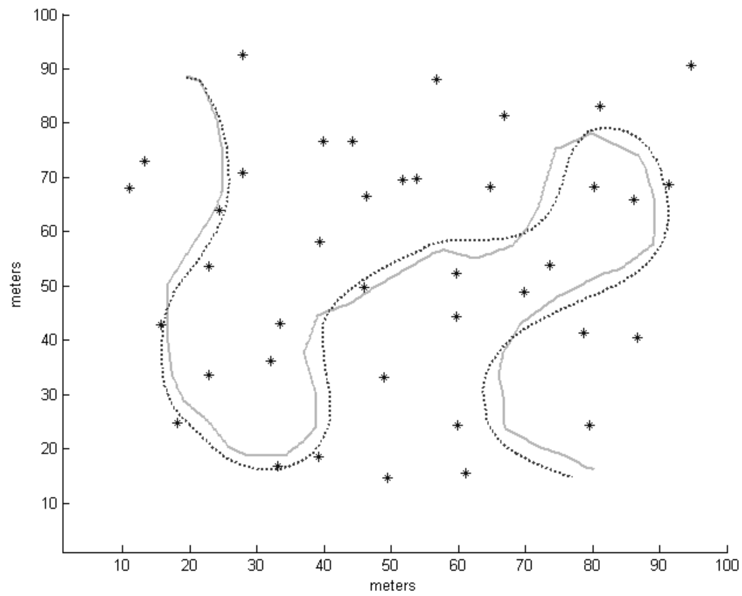

Fig.1. Sample plot from the simulation showing the actual robot trajectory (marked with straight line) and the estimated trajectory (marked with dotted line).

Table 3.1

Measurement results for generated artificial problems

No.	Number of landmarks	Landmark estimation accuracy	State estimation accuracy
1	40	25,67 %	72,19 %
2	80	38,83 %	77,95 %
3	100	84,33 %	82,70 %
4	200	86,16 %	85,61 %

4. CONCLUSIONS

In this paper EKF was presented for state estimation of nonlinear processes. Conducted simulations were focused on typical SLAM scenarios. Achieved results show varying state estimation accuracy that depend on the number of landmarks, with best value of 85,61%.

It is worth to note that by increasing the number of landmarks the state estimation error decreases. However, incorporating more landmarks increases the computational demand, but the increase in estimation accuracy is not guaranteed. Furthermore, one of major drawbacks of EKF is that it can become either pessimistic or optimistic. The error covariance can be bigger than the true one in a pessimistic scenario. On the other hand in an optimistic one the covariance error can be smaller. Given this facts it can be concluded that in both cases the filter can become inconsistent.

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ESTYMACJA STANU W SYMULTANICZNEJ LOKALIZACJI I MAPOWANIU DLA NIELINIOWYCH PROCESÓW PRZY WYKORZYSTANIU ROZSZERZONEGO FILTRU KALMANA

Streszczenie

Praca przedstawia wykorzystanie Rozszerzonego Filtru Kalamana (EKF) w problemie estymacji stanu dla procesów nieliniowych. Szczególny nacisk został położony na użyciu go w algorytmie Symultanicznej Lokalizacji i Mapowania (SLAM). W celu zbadania właściwości EKF przeprowadzono liczne symulacje w typowych dla SLAM scenariuszach postępowania. Wykazano, że algorytm EKF może być użyty do precyzyjnej estymacji stanu w typowym scenariuszu SLAM przy stosunkowo nieznacznym błędzie.