Let's (Bi)Simulate

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1. Introduction
2. Models
3. Bisimulation
4. Bisimulation for probabilistic systems
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- The idea of systems/models closed in a "black box".
- When they are similar, bisimilar or indistinguishable?
- How we can check, if it is the bisimulation?
  ... by playing a game...
- A hold-up for probabilistic systems.

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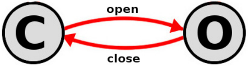
A Transition System

Definition

A transition system is a four-tuple $TS = (S, E, T, s_0)$, where:

- S is the set of states with initial state s_0,
- E is the set of events,
- $T \subseteq S \times E \times S$ is the set of transitions (as usual, the transition (s, a, s_1) is written as $s \xrightarrow{a} s_1$)

[4]



A Nondeterministic Finite Automata

Definition (NFA)

A Nondeterministic Finite Automata is a tuple $NFA = (Q, \Sigma, \delta, q_0, F)$, where:

- Q is the finite set of states, with the start state q_0,
- Σ is the finite set of input symbols,
- δ is the transition function $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \mapsto 2^Q$,
- $F \subseteq Q$ is the set of final (accepting) states.

[1]

Example of NFA

Example

$NFA = (Q, \Sigma, \delta, q, F)$:

$Q = \{q_0, q_1, q_2\}$

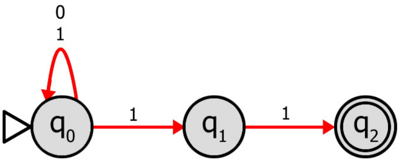
$\Sigma = \{0, 1\}$

$q = q_0$

$F = \{q_2\}$

The transition function

$Q \setminus \Sigma$	0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\emptyset$	$\{q_2\}$
q_2	$\emptyset$	$\emptyset$



A Finite Markov Chain

Definition

A Finite Markov Chain is a pair $MC = (Q, \delta)$, where:

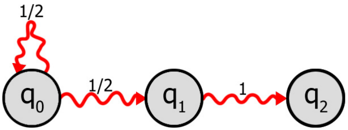
- Q is the set of states,
- δ is the transition function $\delta : Q \mapsto \mathcal{D}(Q)$,

[2]

Notation

If $q \in Q$ and $\delta(q) = P$ with $P(s') = p > 0$, then the Markov chain is said to go from the state s to the state s' with probability p.

Notations: $s \rightsquigarrow P$, $s \overset{p}{\rightsquigarrow} s'$, $\delta(s) = P(s)$, $\delta(s)(s') = p$.



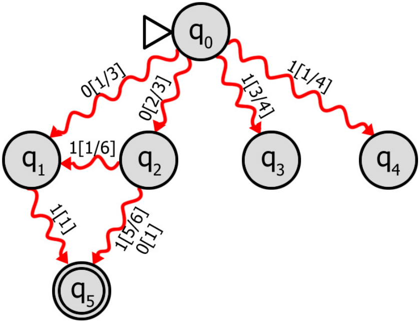
A Finite Reactive Probabilistic Automata

Definition (PA)

A finite reactive probabilistic automata is a tuple $PA = (Q, \Sigma, \delta, q_0, F)$, where:

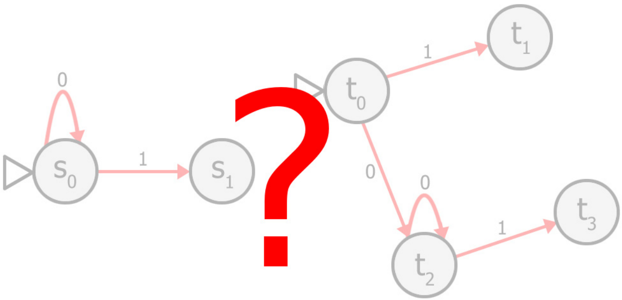
- Q is the finite set of states,
- Σ is the finite set of input symbols,
- $\delta : Q \times \{\Sigma \cup \{\varepsilon\}\} \mapsto \mathcal{D}(Q)$ is the transition function
- $q_0 \in Q$ is the initial state,
- $F \subseteq Q$ is the set of final (accepting) states.

[6]



Bisimulation

- When are two processes (states) behavioraly equivalent?
- What does it mean for two systems to be equal with respect to their communication structures?



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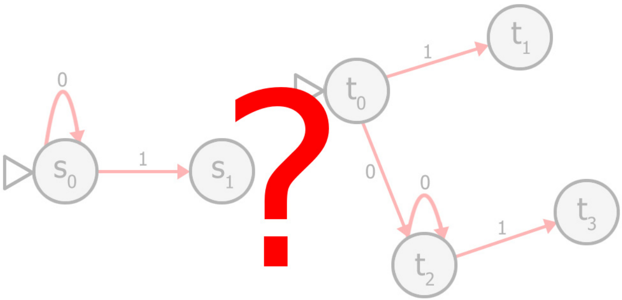

Bisimulation relation

Definition (Bisymulation)

Two transition systems $TS_1 = (S, \Sigma, \delta, s_0)$ i $TS_2 = (T, \Sigma, \delta, t_0)$ are bisimilar iff there is a relation $R \subseteq S \times T$ such that the initial states are related and for all pairs $(s, t) \in R$ and for all $\sigma \in \Sigma$ the following holds:

- whenever $\delta(s, \sigma) = s'$ then there exists $t' \in T$, such that $\delta(t, \sigma) = t'$ and $(s', t') \in R$, and
- whenever $\delta(t, \sigma) = t'$ then there exists $s' \in S$, such that $\delta(s, \sigma) = s'$ and $(s', t') \in R$.

States s, t are called bisimilar, denoted by $s \approx t$.

[5], [6], [3]

Let's play...

...in bisimulation:

- this is a game between two persons: the Player and the Opponent
- the Player tries to prove that systems are bisimilar - the Opponent intends otherwise
- the Opponent opens the game by choosing a transition from the initial state of one of the systems
- the Player have to find an equally labelled transition from initial state of the second system
- new states are starting points for next turn...
- if one of players cannot move - other wins this turn of the game
- the Player loses abundantly, if there are no corresponding transition for Opponent's move
- the Player wins any infinite turn of the game or any repeated configuration

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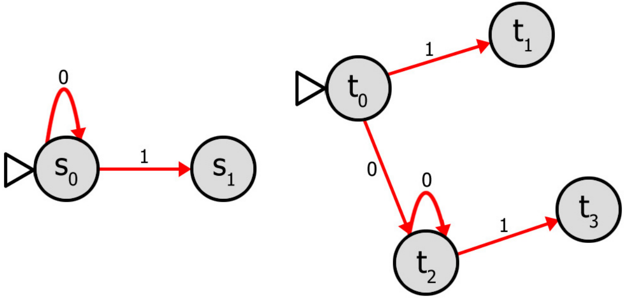
Are they bisimilar?



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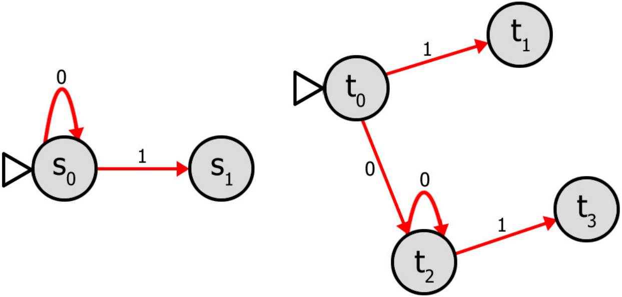

Yes, they are.

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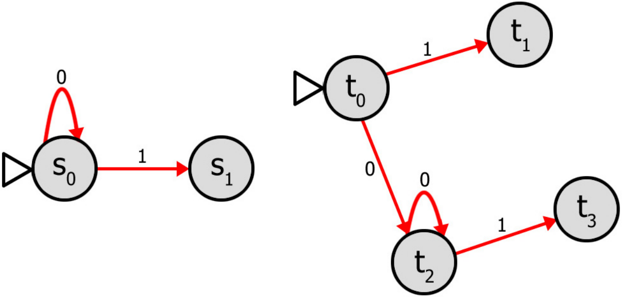

Yes, they are.

$R = \{(s_0, t_0), (s_0, t_2), (s_1, t_1), (s_1, t_3)\}$

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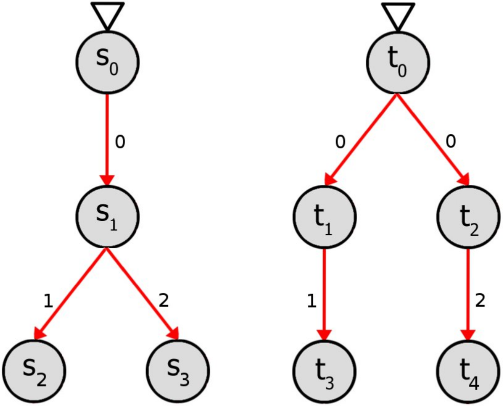

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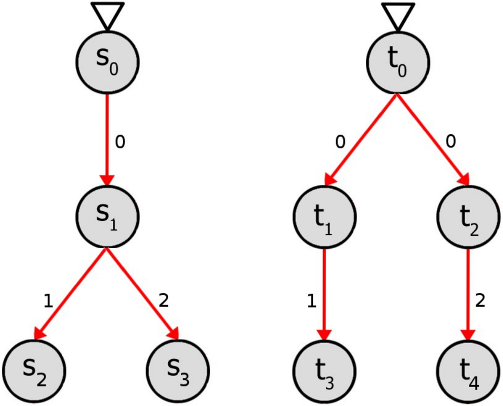

No, they are not.

Introduction

A hold-up for probabilistic systems.

- How to collate probability distributions?
- What to do with read symbols?
- Is it game still the same?

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Equivalence Relation

Proposition

Let R be an equivalence relation on the set S and let $P_1, P_2 \in \mathcal{D}(S)$ be probability distributions. Then:

$$P_1 \equiv_R P_2 \iff \forall C \in S_{/R} : P_1[C] = P_2[C], \tag{1}$$

where C is an abstract class. [6]

Definition

Let R be an equivalence relation on the set S, A a set, and let $P_1, P_2 \in \mathcal{D}(S)$ be probability distributions. Define:

$$P_1 \equiv_{R,A} P_2 \iff \forall C \in S_{/R}, \forall a \in A : P_1[a, C] = P_2[a, C], \tag{2}$$

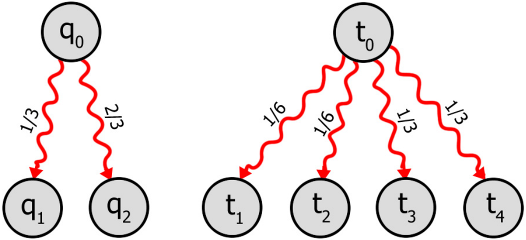
[6]

Bisimulation for Markov Chains

Definition

Equivalence relation on the set of states Q of Markov chain (Q, δ) will be a bisimulation relation iff $\forall (q,t) \in R$:

$$\delta(q) = P_1, \quad \text{then exists} \quad \delta(t) = P_2 \quad \text{such, that} \quad P_1 \equiv_R P_2. \tag{3}$$



$$R = \{(q_0,t_0),(q_1,t_1),(q_1,t_2),(q_2,t_3),(q_2,t_4)\}$$

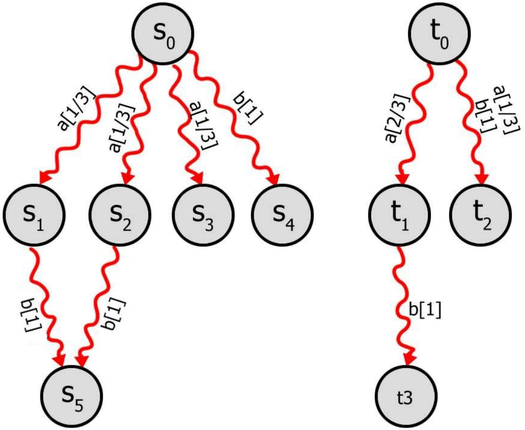
Bisimulation for PA I

Definition

Let $PA_1 = (S, \Sigma, \delta)$ and $PA_2 = (T, \Sigma, \delta)$ be two probabilistic automatas, then exists a bisimulation relation $R \subseteq S \times T$, if for all pairs $(s, t) \in R$ and for all $\sigma \in \Sigma$ holds:

- if $\delta(s, \sigma) = P_1$ then exists a probability distribution P_2 such, that for $t' \in T$ exists $\delta(t, \sigma) = P_2$ and $P_1 \equiv_{R,\Sigma} P_2$. [6]

Bisimulation for PA II



$$R = \{(s_0, t_0), (s_1, t_1), (s_2, t_1), (s_3, t_2), (s_4 3, t_2), (s_5, t_3)\}$$

Summary

- A bisimulation relation is a tool for finding equivalent systems.
- A game as a simple way for checking, if it is a bisimulation.
- For probabilistic systems it is much more complicated...

- Bisimulation as a foundation for "stronger" relations, useful in, for example, minimization of systems.

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